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Determination of Third- and Fourth-Order Longitudinal Elastic Constants by Shock Compression Techniques—Application to Sapphire and Fused Quartz*

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A number of solids sustain large elastic compressions under shock-wave loading. In these solids, measurements of the stress and compression in the direction of shock propagation can be used to calculate both third- and fourth-order longitudinal elastic constants if measurements are carried out over a wide range of compressions. Only limited measurements of fourth-order constants have been previously determined by other techniques. Determinations of third-order constants under these large elastic compressions afford the opportunity to test the applicability of the finite-strain formulation of constitutive relations. A general method for calculating these third- and fourth-order constants is presented and applied to shock compression data for sapphire and fused quartz. For sapphire, it is found that $C_{111} \approx C_{333} = -(3.3 \pm 0.3) \times 10^4$ kbar and $C_{1111} \approx C_{3333} = +(5.0 \pm 1.5) \times 10^5$ kbar. For fused quartz, it is found that $C_{111} = +(5.5 \pm 0.1) \times 10^3$ kbar and $C_{1111} = +(110 \pm 10) \times 10^3$ kbar. The technique and method of analysis seem generally applicable to solids that exhibit elastic limits of a few percent of their longitudinal elastic constants.

INTRODUCTION

When subjected to shock-wave compression, a number of solids are observed to exhibit unusually large elastic limits. Noting that large elastic compressions can be achieved under shock compression, Fowles[1] expressed the finite-strain high-order elastic constant theory in terms suitable for analysis of elastic shock-compression data. He proposed that longitudinal fourth-order elastic constants could be computed from shock-compression data if the second- and third-order elastic constants were known. From this analysis, the longitudinal fourth-order elastic constants of α quartz were computed from the shock-compression data in the elastic range, i.e., below the Hugoniot elastic limit. The present paper extends the analysis of shock-compression data to the determination of both third- and fourth-order longitudinal elastic constants.

A number of measurements of third-order elastic constants have been accomplished with static compression techniques, including measurements on Ge,[2–6] MgO,[2] Si,[4,6] fused quartz,[2] α quartz,[7] and sapphire.[8] Measurements of these third-order constants are of both fundamental and applied interest. The third-order constants are associated with anharmonicity of a crystal lattice; hence, they may be used to calculate generalized Grüneisen parameters.[9] Furthermore, quantitative descriptions of acoustic amplification at microwave frequencies in solids[10,11] requires knowledge of the third-order elastic constants. If piezoelectric solids are used for amplification, high-order piezoelectric constants are also important.[12–14] The attenuation in microwave delay lines is influenced by the Akhiezer phonon–phonon interaction mechanism,[15] which can be calculated from the third-order elastic constants.

Only a limited number of fourth-order elastic constant measurements have been accomplished and there is no established technique for their determination. In addition to Fowles's measurements, fourth-order constants of several cesium halides have been measured by ultrasonic techniques,[16] and several combinations of fourth-order constants of fused quartz have been determined in uniaxial tension experiments.[17] Fourth-order constants have not yet been required for interpretation of microwave phenomena, but it has been suggested that harmonic generation in stressed crystals could be used to determine fourth-order constants.[18]

Although, at present, only longitudinal elastic constants can be determined from shock-compression measurements, it appears that these longitudinal constants are often of interest. Furthermore, the determination of the third-order constant under large compressions permits a test of the formulation of the finite-strain theory.

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compressions of 0.5%. Between 0.5% and 2.5%, the third-order constant gives a significant contribution, while the fourth-order constant gives a significant contribution for compressions from 2.5% to 6%.

IV. CONCLUSION

Analysis of the sapphire and fused-quartz shock-compression data demonstrates that the analytical method employed permits the determination of both third- and fourth-order longitudinal elastic constants. Although the third-order constants may be determined by other techniques, the fourth-order constants have been determined only by shock-compression techniques. The method is limited to solids that sustain large elastic compressions in uniaxial strain; however, a number of solids exhibit large Hugoniot elastic limits. The materials with known large Hugoniot elastic limits[38] include sapphire, α quartz, MgO, Ge, Si, CdS, InSb, TiO_2, B_4C, BeO, and yttrium iron garnet. Thus, the method may be applied to a reasonably large number of solids of technical interest. Although shock-compression measurements have been performed on all these solids, the measurements are usually limited to several discrete stress–volume points, and these data are insufficient for the determination of third- and fourth-order constants.

Even though there is some question as to the appropriateness of extending the theory to compression data to fourth order,[8] it is clear that the experimentally observed compressions of α quartz, sapphire, and fused quartz can be adequately described by the fourth-order constant development. Furthermore, it appears that shock-compression measurements can play a generally useful role in the determination of longitudinal third- and fourth-order elastic constants. If precise stress-versus-volume relations can be obtained under large elastic compressions, it appears that the [illegible] formulations can be given an evaluation. The present measurements are somewhat limited in accuracy, but it appears that the stress–strain formulation given here gives an appropriate description to both the ultrasonic and shock-compression data of sapphire and fused quartz.

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